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One-Level, Two-Point Method for Estimation of Thermal Diffusivity by the Converging Thermal-Wave Technique

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The thermal diffusivity of several kinds of metal sheets was measured by the converging thermal-wave technique. This is a typical technique which can obtain the in-plane thermal diffusivity by measurement of the temperature evolution at the center of the pulsed annular laser beam when the laser beam irradiates the surface of the samples. However, the rapid temperature increase and nonlinearity of the infrared detector in the earlier part, and convection heat loss from the sample surface in the later part, of the temperature evolution can be sources of errors. In this study, when the temperature of the center reached half of the maximum, the times t_1 and t_2, in the ascending and descending parts of the temperature evolution curve, respectively, were determined and other points, t'_1 and t'_2, were determined by increasing the temperature to 90% of the maximum in successive increments of 10% of the maximum. By using these determined times, the variation of thermal diffusivity was investigated and compared with results from existing methods and with reference values.

KEY WORDS: convection heat loss; converging thermal-wave technique; flash method; infrared detector; metal sheet; thermal diffusivity.

1. INTRODUCTION

Thermal diffusivity is a physical parameter that provides both direct and indirect information on materials of industrial interest. Direct knowledge of thermal diffusivity is required, for example, for fire-resistant ceramic

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coatings, insulating foams, or modeling of cooling in polymeric molding. Indirect information provided by thermal analysis is a powerful tool for material characterization. Recently, various kinds of thin-film or thin-plate materials have been developed in many fields, for example, for use in advanced microelectonic devices and in space and atomic technologies [1].

There are many thermal diffusivity measurement methods and a proper method is usually selected by considering the specimen size, shape, characteristics, and temperature range. The flash method developed by Parker et al. [2] has been widely used due to the need for rapid and simple measurements. This flash method is used mainly to measure the through-plane thermal diffusivity [3]. The converging thermal-wave technique, developed by Cielo et al. [4] and by Lu and Swann [5], is a kind of flash method which can measure the in-plane thermal diffusivity by the measurement of temperature evolution at the front and rear center of a pulsed annular laser beam, when the laser beam irradiating the surface of the samples and the thermal wave converge in the center.

In the conventional converging thermal-wave techniques, the measurement of thermal diffusivity is not easy. The thermal diffusivity is obtained by checking the maximum temperature time [5] or half-maximum temperature time [4, 6, 7]; however, the reduction of errors caused by the rapid temperature rise in the earlier evolution curve, the relatively wide range of maximum time, and the convection heat loss from the sample surface complicate the measurement. Therefore, despite the same experimental data, the thermal diffusivities obtained show scattered values, which depend on the data analysis method.

The purpose of this paper is to describe a new method which is expected to be useful for thermal diffusivity measurements. The method is based on determining the times of equal temperatures on the ascending and descending parts of the temperature evolution curve. These times are subsequently used to calculate the thermal diffusivity. The proposed method makes it possible to reduce the errors caused by several factors mentioned above.

2. THEORY

In the case of an infinitely thin and infinitely short annular laser pulse of energy E liberated per unit thickness of a thin sample, the time dependent temperature evolution $T(t)$ obtained at the center of the annulus of radius R, under the assumption that the finite pulse time is neglected, is [1]

$$T(t) = \frac{E}{c\rho\pi R^2} \frac{R^2}{4\alpha t} \exp\left(-\frac{R^2}{4\alpha t}\right) \tag{1}$$

where $\alpha = \lambda/(c\rho)$ is the thermal diffusivity of the material, λ is the thermal conductivity, c is the specific heat at constant pressure, and ρ is the mass density. This expression can be used to evaluate the thermal diffusivity α from the experimentally observed temperature evolution curve.

Except for the detection method of temperature evolution, Cielo et al. [4] and Lu and Swann [5] performed the same experiment. Cielo et al. detected the front surface temperature evolution of the specimen by what is called the reflection method; however, to avoid the errors caused by direct incidence of a stray laser beam, Lu and Swann detected the temperature evolution at the rear surface by the transmission method. If it is assumed that the specimen is very thin, then there is no temperature gradient along the thickness direction and there is no strong stray light. In this case the two experiments can be treated as one.

Figure 1 shows the theoretical curve obtained from Eq. (1). Cielo et al. [4] defined the half-maximum time $t_{1/2}$ when the temperature evolution becomes half-maximum, and they expressed the thermal diffusivity as

$$\alpha = \frac{R^2}{10.7 t_{1/2}} \tag{2}$$

Lu and Swann [5] differentiated Eq. (1) with respect to t, obtained the maximum temperature time t_{m} when the temperature evolution becomes maximum, and expressed the thermal diffusivity as

$$\alpha = \frac{R^2}{4 t_{\mathrm{m}}} \tag{3}$$

However, these expressions neglect several factors, such as the finite pulse time effect [8] and convection heat loss from the surface of the sample, which cause errors [9].

Here we present a method for this procedure which may be called the "one-level, two-point method." Let us draw a line parallel to the time axis. The intersection points of this parallel line with the temperature evolution curve will give us characteristic points in Fig. 1, where the equality of temperatures occurs at the corresponding moments of time t_1 and t_2, i.e.,

$$\frac{E}{c\rho\pi R^2}\frac{R^2}{4\alpha t_1}\exp\left(-\frac{R^2}{4\alpha t_1}\right) = \frac{E}{c\rho\pi R^2}\frac{R^2}{4\alpha t_2}\exp\left(-\frac{R^2}{4\alpha t_2}\right) \tag{4}$$

After elementary transformations, using Eq. (4), we can now obtain Eq. (5).

$$\alpha = \frac{R^2(t_2 - t_1)}{4 t_1 t_2 \ln(t_2/t_1)} \tag{5}$$

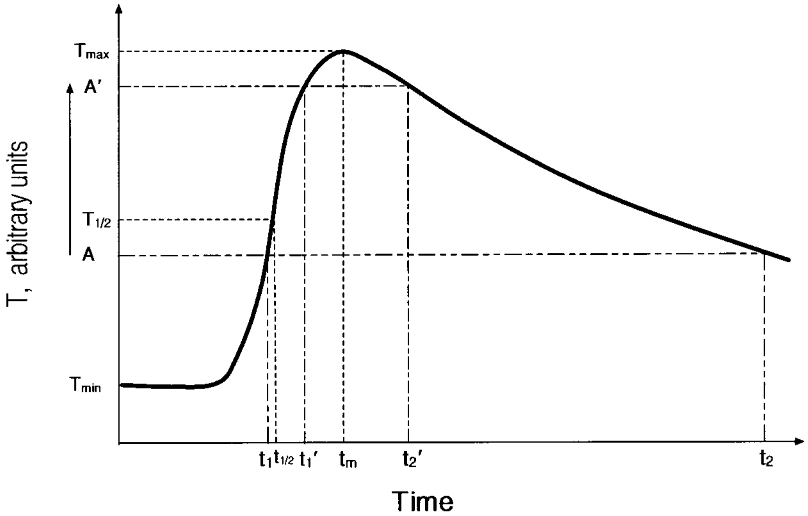

Fig. 1. Theoretical curve of temperature evolution and definition of the characteristic points.

Therefore, if we get t_1 and t_2 from the temperature evolution curve, we can obtain the thermal diffusivity of the sample. Because of the nonlinearity of the infrared detector [10, 11], the rapid increase in the temperature evolution curve in the earlier part results in values lower than the true values, and convection heat loss from the surface in the later part after the peak point produces thermal diffusivity values higher than the true values [12]. Thus, the trend of errors can be estimated with the investigation of thermal diffusivity variation by monitoring $T(t)/T_{\max}$ closer to the peak of the temperature evolution curve and determining times t_1' and t_2', as shown in Fig. 1.

3. EXPERIMENTAL

Figure 2 is a schematic diagram of the experimental apparatus for the converging thermal-wave technique, which is similar to the Lu and Swann [5] transmission method. The pulsed laser beam (duration, 0.8 ms; energy, 1.8 J) from an Nd:glass laser becomes annular when it passes through the combination of a converging lens and an axicon and is focused on the sample surface. The reflectivity of the metal surface is higher than 80%, thus, to reduce that reflectivity the surface was coated with a liquid graphite thin film less than 100 nm thick. The axicon was made of fused

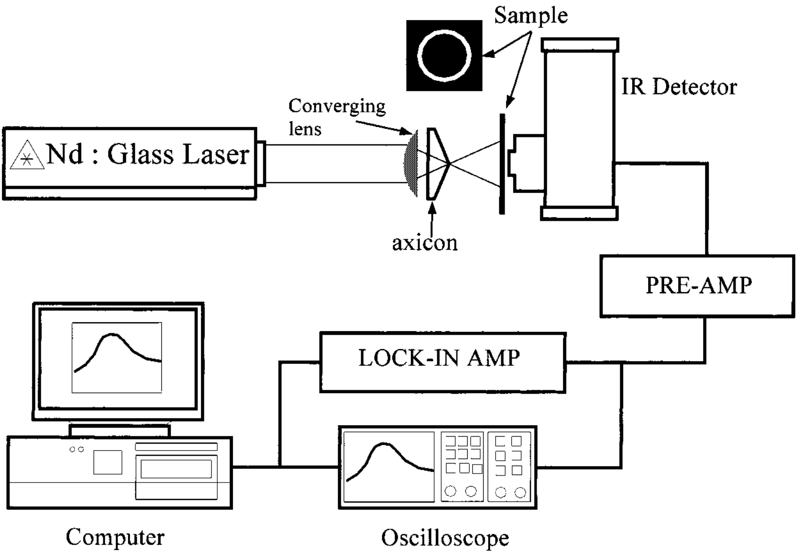

Fig. 2. Schematic diagram of the experimental apparatus for the converging thermal-wave method.

quartz, and its apex angle was 170°. The radius and width of the annular laser beam can be calculated by Snell's law by using the values of the focal length of the converging lens and the length from the lens to the axicon. The converging time of the thermal wave depends on the radius of the annular laser beam, so it must be maintained at between 4 and 7 mm for optimum results [4].

The thermal wave converted from the annular laser beam propagates in all directions, and the temperature evolution of the center is detected in the rear surface of the sample by an infrared HgCdTe detector. The temperature evolution was recorded by a digital oscilloscope through a GPIB interface, and the thermal diffusivities could be calculated with changes in $T(t)/T_{\max}$ to larger values by substituting the obtained values of t'_1 and t'_2 into Eq. (5).

The four kinds of metal samples measured were aluminum, copper, nickel, and stainless steel 304. Table I summarizes the sample materials. For the above formulations to be valid, thin specimens must be used. Therefore, the thickness of all samples was about 0.08 mm, much smaller than the radius of the annular laser beam [9]. Also, to remove the effect of the diverging beam reflected from the edge of the sample, the sample was cut to 45 × 45 mm.

Table I. Sample Descriptions

Specimen	Purity (%)	Thickness (mm)
Aluminum	99.5	0.075
Copper	99.9	0.080
Nickel	99.7	0.080
Stainless steel 304	Cr, 18 to 20; Ni, 8 to 11; Mn, <2; Si, <1; balance, Fe	0.070

4. RESULTS AND DISCUSSION

Figure 3 shows the temperature evolution curve at the rear-side center of the laser beam after irradiating the annular laser beam on the aluminum sample. It shows the same trend as expected from Eq. (1), that is, immediately after laser irradiation, the temperature increases rapidly, and after the peak, it decreases slowly.

The trends in other samples such as copper, nickel, and stainless steel 304 are the same as that in aluminum. The thermal diffusivities of four kinds of samples obtained by changing the $T(t)/T_{\max}$ values from 50 to 90% in successive 10% increases are shown in Tables II–V, compared with the values calculated by the equations of Cielo et al. [4] and Lu and

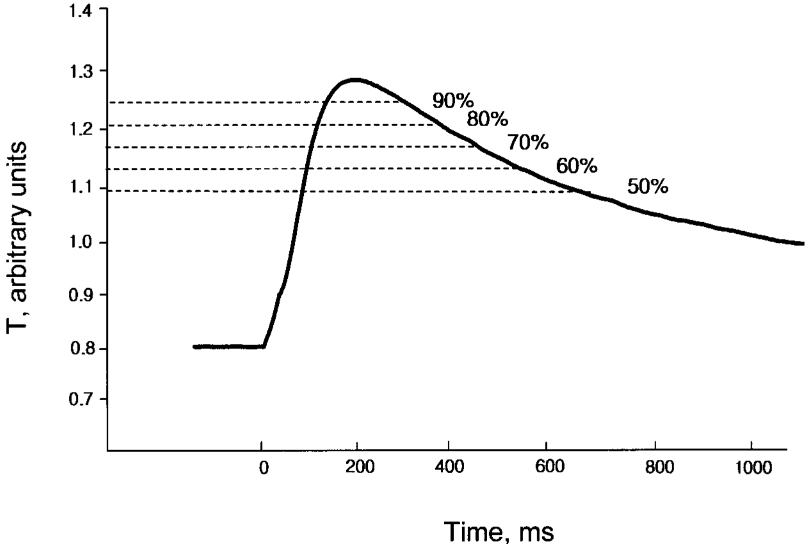

Fig. 3. Temperature evolution curve at the center of the annular laser beam on an aluminum sheet sample.

Table II. Thermal Diffusivities of Aluminum Samples Obtained by Eq. (5) Compared to Values from the Lu and Cielo Equations and to the Reference Value ($R = 7.25$ mm, $t_m = 133.34$ ms, $t_{1/2} = 66.65$ ms)

			Thermal diffusivity ($10^{-4}\,m^2 \cdot s^{-1}$)			
$T(t)/T_{max}$ (%)	t_1 (ms)	t_2 (ms)	$\frac{R^2(t_2-t_1)}{4t_1t_2\ln(t_2/t_1)}$	$\frac{R^2}{4t_m}$ [5]	$\frac{R^2}{10.7t_{1/2}}$ [4]	Reference value [12]
90	88.87	220.70	0.971	0.986	0.737	0.971
80	73.24	284.67	0.982			
70	62.99	352.54	0.995			
60	54.69	436.52	1.012			
50	47.36	554.20	1.032			

Table III. Thermal Diffusivities of Copper Samples Obtained by Eq. (5) Compared to Values from the Lu and Cielo Equations and to the Reference Value ($R = 7.04$ mm, $t_m = 99.12$ ms, $t_{1/2} = 49.56$ ms)

			Thermal diffusivity ($10^{-4}\,m^2 \cdot s^{-1}$)			
$T(t)/T_{max}$ (%)	t_1 (ms)	t_2 (ms)	$\frac{R^2(t_2-t_1)}{4t_1t_2\ln(t_2/t_1)}$	$\frac{R^2}{4t_m}$ [5]	$\frac{R^2}{10.7t_{1/2}}$ [4]	Reference value [12]
90	68.36	173.83	1.177	1.248	0.933	1.17
80	56.15	227.05	1.187			
70	48.34	281.25	1.204			
60	41.99	347.66	1.226			
50	35.65	437.01	1.272			

Table IV. Thermal Diffusivities of Nickel Samples Obtained by Eq. (5) Compared to Values from the Lu and Cielo Equations and to the Reference Value ($R = 7.67$ mm, $t_m = 620.12$ ms, $t_{1/2} = 310.06$ ms)

			Thermal diffusivity ($10^{-5}\,m^2 \cdot s^{-1}$)			
$T(t)/T_{max}$ (%)	t_1 (ms)	t_2 (ms)	$\frac{R^2(t_2-t_1)}{4t_1t_2\ln(t_2/t_1)}$	$\frac{R^2}{4t_m}$ [5]	$\frac{R^2}{10.7t_{1/2}}$ [4]	Reference value [12]
90	417.48	1069.34	2.284	2.372	1.774	2.30
80	346.68	1372.07	2.305			
70	295.41	1689.46	2.357			
60	253.91	2131.35	2.399			
50	217.29	2780.77	2.448			

Table V. Thermal Diffusivities of Stainless Steel 304 Samples Obtained by Eq. (5) Compared to Values from the Lu and Cielo Equations and to the Reference Value ($R = 4.95$ mm, $t_m = 1490.0$ ms, $t_{1/2} = 750.0$ ms)

$T(t)/T_{\max}$ (%)	t_1 (ms)	t_2 (ms)	Thermal diffusivity ($10^{-6}\,\text{m}^2 \cdot \text{s}^{-1}$) $\frac{R^2(t_2-t_1)}{4t_1t_2\ln(t_2/t_1)}$	$\frac{R^2}{4t_m}$ [5]	$\frac{R^2}{10.7t_{1/2}}$ [4]	Reference value [12]
90	1165	2290	3.821	4.109	3.052	3.95
80	925	2810	3.996			
70	805	3450	4.007			
60	685	4410	4.055			
50	605	5650	4.045			

Swann [5] and with reference data [12]. The values obtained when $T(t)/T_{\max}$ was lower than 50% are excluded because of the relatively low signal-to-noise ratio.

For the case of aluminum samples in Table II, as the value of $T(t)/T_{\max}$ increases from 50 to 90%, the thermal diffusivity decreases slightly, from 1.032×10^{-4} to $0.971 \times 10^{-4}\,\text{m}^2\,\text{s}^{-1}$, about 6%, and the lowest value agrees well with the reference value. The thermal diffusivity of copper, nickel, and stainless steel 304 shows similar trends, and the thermal diffusivity obtained when $T(t)/T_{\max}$ is 80 to 90% agrees well with the reference values.

The thermal diffusivity obtained by the Lu and Swann equation agrees with the reference data [12] within several percent, but the diffusivity calculated by the equation of Cielo et al. shows 20% lower values. This is caused by the fact that the temperature evolution rate in the peak region is relatively slower than that in the other region, thus t_m can be determined accurately. However, because of the nonlinearity of the infrared detector [11], the $t_{1/2}$ obtained in the earlier part of the temperature evolution curve becomes longer than real time and contains errors caused by the rapid evolution. These were verified by the fitting of experimental results to the theory. The obtained results in Tables III–V show trends similar to that of the aluminum sample.

If these error factors are included, all the thermal diffusivities obtained for one sample must be the same regardless of different values of $T(t)/T_{\max}$. But the fact is that the thermal diffusivity decreases as the $T(t)/T_{\max}$ value increases, caused by the increase in detector nonlinearity and to a lesser degree, by the increase in convection heat loss. It follows from the experimental results that the 80 to 90% rise in $T(t)/T_{\max}$ can be recommended for calculation of the thermal diffusivity, because the data obtained at these time windows have minimal errors and show the best agreement with the reference values.

The pulse duration of the laser beam is 0.8 ms, which is very short compared to the several hundred milliseconds for the temperature evolution. Therefore, the finite pulse time effect can be neglected. The pulse energy is 1.8 J, and the amount of temperature increase at the center by this energy is less than 10 K. Therefore, the nonlinearity of the infrared detector is about 3% [4]. To reduce this effect, a lower-energy pulse laser beam is recommended, but the signal-to-noise ratio decreases simultaneously. Therefore, a higher-quality main amplifier must be used. Also, to avoid problems of convection heat loss, the sample must be put into a vacuum chamber, hence, the signal level will be reduced. On the contrary, a better signal, not affected by convection, should be obtained since radiation travels best in a vacuum.

5. CONCLUSIONS

The in-plane thermal diffusivities of four kinds of metal samples, obtained by changing the $T(t)/T_{\max}$ values from 50 to 90% in successive 10% increases, show that the thermal diffusivities decrease slightly in comparison with the true values. The phenomenon is thought to be caused by the slight increase in nonlinear effects in the infrared detector and the simultaneous increase in convection heat loss. It follows from the experimental results that the 80 to 90% rise in $T(t)/T_{\max}$ can be recommended for calculation of the thermal diffusivity, because the data thus obtained at this moment have minimal errors and show the best agreement with the reference values.

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REFERENCES

1. S. H. Lee, S. W. Kim, and S. W. Park, *Korean Appl. Phys.* **3**:393 (1990).
2. W. J. Parker, R. J. Jenkins, C. P. Butler, and G. L. Abbott, *J. Appl. Phys.* **32**:1679 (1961).
3. S. W. Kim and R. E. Taylor, *Int. J. Thermophys.* **14**:135 (1993).
4. P. Cielo, L. A. Utracki, and M. Lamontagne, *Can. J. Phys.* **64**:1172 (1986).
5. G. Lu and W. T. Swann, *Appl. Phys. Lett.* **59**:1556 (1991).
6. E. Worner, C. Wild, W. Muller-Sebert, M. Funer, M. Jehle, and Koidl, *Rev. Sci. Instrum.* **69**:2105 (1998).

7. H. Park, J. S. Hong, Y. J. Han, and H. B. Chae, *J. Nat. Sci. Soonchunhyang* **5**:77 (1999).
8. T. Azumi and Y. Takahashi, *Rev. Sci. Instrum.* **52**: 1411 (1981).
9. F. Enguehard, D. Boscher, A. Deom, and D. Balageas, *Mater. Sci. Eng*. **B5**:127 (1990).
10. S. W. Kim, S. H. Hahn, J. C. Kim, D. Chi, and J. H. Kim, *Korean Appl. Phys.* **7**:46 (1994).
11. J. J. Hoefler and R. E. Taylor, *Int. J. Thermophys*. **11**:1099 (1990).
12. F. P. Incropera and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 2nd ed. (John Wiley, New York, 1990), Appendix A.